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PRELIMINARY STUDY OF BENEFITS OF EARLY CORRECTIONS AND RCS VERNIER VELOCITY CONTROL FOR APOLLO MIDCOURSE VELOCITY

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SUMMARY

Preliminary results are reported for two problems significant in the Apollo midcourse velocity correction requirement. The effects of making early velocity corrections and a comparison of SPS and RCS cutoff error penalties are studied. It is concluded that an RCS vernier correction capability provides a very significant improvement in performance compared to SPS alone cutoff characteristics, and if the velocity correction errors are small, early corrections also provide performance gains.

INTRODUCTION

The use of MSFN navigation data for calculation of the initial translunar and transearth guidance corrections offers the possibility of making these corrections earlier than if star-landmark or similar optical data is the sole navigational source. If the midcourse fuel consumption is to be predicated on the basis of early corrections, the ground rule that the mission will be aborted if the early ground tracking is not available must be made. This study is intended to estimate the fuel benefits of early corrections in order to provide a basis for establishing or discarding this ground rule.

A second area that is problematic for the guidance system is the large velocity cutoff error of the SPS system during the transearth phase of the mission. The RCS system can conceivably be used as a vernier control to alleviate this problem. A comparison of the fuel cost with and without this vernier capability is included.

PROBLEM MECHANIZATION

A linearized description of the dynamics of deviations from a reference trajectory is used to provide the statistical analysis of velocity correction requirements. This technique is essentially identical to that used by Battin in MIT Instrumentation Laboratory Report R-341. The digital computer mechanization used to generate data for this note allows a variation from Battin's formulation, in that the residual error after a velocity correction can be represented by a constant variance in the magnitude of the cutoff velocity as well as by a variance proportional to the magnitude of the correction. A complete mathematical development of the technique used in this study is contained in the appendix.

The current version of this program operates under the assumption that the navigation information is exact. This should be a reasonable representation for the MSFN network, since the errors associated with state estimation are considerably less than the errors associated with the velocity correction system.

SCOPE OF THE INVESTIGATION

Two problems were considered in this investigation, evaluation of the benefits of an early correction capability, and evaluation of the penalties associated with the large SPS velocity cutoff errors in the transearth phase.

In order to investigate the early correction benefits, a basic correction schedule based on the MIT nominal was assumed. This schedule makes the first outbound correction at 8 hours and the first transearth leg correction at 14.5 hours from pericynthion. It was assumed that a good estimate of the state could be obtained using ground data within two hours from both translunar injection and transearth injection. Accordingly, an early correction schedule based on this assumption was compared with the nominal.

Errors in velocity correction application come from three major sources. The first is net pointing error due to platform misalinement, thrust vector misalinement and steering dynamics. The variance in pointing errors was assumed to be .01 rads, a value consistent with MIT calculations. There are two contributions to errors in magnitude, the accelerometer quantization level and the uncertainty in thrust tail-off. The accelerometer quantization level is 5.85 cm/sec, or .19 ft/sec. For the SPS, the tail-off uncertainty was assumed to be 1000 lb-secs (based on the SPS specification), leading to velocity errors of .36 ft/sec for the translunar leg (90,000# vehicle) and 1.65 ft/sec transearth (19,500# vehicle). The RCS tail-off uncertainty is well below the quantization level of the accelerometer, and can be ignored in comparison to it. The resulting RSS velocity correction magnitude errors for the SPS are .40 ft/sec translunar and 1.66 ft/sec transearth, and the RCS vernier capability is .19 ft/sec throughout.

The velocity correction magnitude error was first assumed to be directly proportional to the magnitude of the velocity correction. A scale factor of .01 was used for consistency with MIT results in this area. In the case where the velocity corrections are small, this leads to unreasonable accuracies for the system. Accordingly, the statistics of a constant variance in magnitude error were introduced, and the cutoff precisions listed above were used.

A single circumlunar trajectory was used to generate the transition matrices for this investigation. This was a typical trajectory with pericynthion 70.6 hours from launch and perigee 69.3 hours after pericynthion. The trajectory was generated using the Encke integration scheme and precise ephemerides for the sun and moon.

RESULTS AND DISCUSSION

A uniform set of errors in initial conditions at translunar launch was used in all cases, and is presented in Table I. These errors are consistent with those assumed at MIT, as presented in S.G.A. Memo #31.

The results are presented in Table II. The first three cases are reproduced from MIT SGA Memo #31 (revised) for comparison with local work. Case 1 used a 10 arc second accuracy sextant as the sole navigation information source. Case 2 used the sextant plus radar range-rate at 15 minute intervals near earth and moon, otherwise at 30 minute intervals (128 points translunar, 150 transearth). Case 3 used both radar range and range-rate at 15 minute intervals near earth and moon, otherwise at 30 minute intervals. A review of these three runs illustrates the advantages of radar information, with a drop in corrective velocity increment from 148.7 ft/sec to 51.8 ft/sec. A minor point is the advantage that the combined information in case 2 gives on the out-bound leg and the unexplained penalty inbound. The Kalman filter is so constructed that information is weighted by its precision before incorporation into the estimate, and hence in theory adding optical data to radar data should not degrade the estimate. The non-linearity of this problem or some inconsistency in running the case must account for this phenomenon.

Case 4 is a comparison of the local results to the MIT program. This illustrates the effects of the assumption of perfect information, the fuel requirement for this case being roughly 60% of the corresponding case 3 with imperfect information. The fact that the difference is not an order of magnitude or more is the basis for the assertion that the local results are significant with appropriate adjustments in magnitude.

Case 5 is the same as case 4 except that the first translunar correction is made at 1.5 hours rather than at 8 hours from boost cutoff, and the first transearth correction is made 2 hours after pericynthion rather than at 14.4 hrs. The savings in corrective velocity requirement is significant (28%) and can be attributed chiefly to the first translunar correction improvement. This is logical, since the vehicle is moving out of a strong gravitational field at this time and errors propagate rapidly under these conditions.

Inspection of the second and third corrections both outbound and inbound reveals that they are so small as to be impractical. Moreover, the fact that the errors in the velocity correction are scaled to its magnitude causes the residual errors after these corrections to be infinitesimal. In order to get a more realistic result, velocity cutoff errors with a constant variance in magnitude were represented.

Cases 6 and 7 are constant magnitude variance results consistent with the SPS cut-off conditions for late and early first corrections respectively. The realistic cut-off errors amplify the velocity requirement considerably. Comparison of the early and late correction cases reveals that the advantage of the early correction has decreased, since the early correction transearth actually costs more than the late correction. This is attributed to the fact that the cut-off errors at transearth boost are now comparable in magnitude to the errors in velocity corrections, and the first correction is made so early that it merely replaces one set of errors with another of like magnitude. In fact, the results indicate that even the late first correction case would benefit from further delay. A combination of early first correction translunar and late first correction transearth would give a 6.7% improvement.

Cases 8 and 9 are again constant magnitude results assuming that RCS vernier correction capability exists. The smaller cut-off errors give much better results, yielding nearly 60% improvement. This indicates the extreme desirability of having the vernier capability. A comparison of early and late correction schedules indicates that the early correction is more profitable with the RCS cut-off accuracy, providing a 19% improvement.

CONCLUSIONS

This note reports a preliminary investigation of Apollo mid-course velocity correction requirements, in particular covering the benefits of early correction capability and the relative cost of SPS and RCS cut-off errors. A linearized representation of the problem was used with the assumption that perfect navigation information was available, and the capability of using either a proportional or constant statistical variance in the magnitude of the velocity correction.

In summary, it is asserted that the cases run with the assumption of perfect information provide significant qualitative results. The proportional variance in velocity correction magnitude is unrealistic for cases of this type. The SPS cut-off characteristics cause a heavy penalty in performance, which can be alleviated by use of RCS vernier correction capability. If residual errors in the velocity corrections are small, an early correction capability provides significant savings in velocity requirements.

TABLE I

INITIAL CONDITION	DEVIATIONS
1. Translunar	
δR Altitude, ft.	1850.
δR Range	2270.
δR Track	1372.
δV Altitude, ft/sec	8.33
δV Range	3.21
δV Track	3.58
2. Transearth	
δR Altitude, ft.	370.
δR Range	1110.
δR Track	897.
δV Altitude, ft/sec	2.43
δV Range	.84
δV Track	2.35

VELOCITY CORRECTION TIME SCHEDULES

Correction Number		Late	Early
1.	at	8. hrs.	1.5 hrs.
2.	at	57. hrs.	57. hrs.
3.	at	69.6 hrs.	69.6 hrs.
4.	at	85. hrs.	72.6 hrs.
5.	at	130. hrs.	130. hrs.
6.	at	139. hrs.	139. hrs.

TABLE II

Midcourse V Requirements

CASE	1	2	3	4	5	6	7	8	9
Pointing Error (Rads)	.01	.01	.01	.01	.01	.01	.01	.01	.01
Scale Factor ($\delta\hat{V}/\Delta V$)	.01	.01	.01	.01	.01	---	---	---	---
$\delta\hat{V}$ Translunar (ft/sec)	---	---	---	---	---	.40	.40	.19	.19
$\delta\hat{V}$ Transearth	---	---	---	---	---	1.65	1.65	.19	.19
Information	1σ Sextant	MSFN + Sextant	MSFN	Perfect	Perfect	Perfect	Perfect	Perfect	Perfect
First Corrections	Late	Late	Late	Late	Early	Late	Early	Late	Early
ΔV_1 (ft/sec)				22.43	15.21	22.43	15.21	22.43	15.21
ΔV_2				1.75	1.98	2.84	4.96	1.50	2.60
ΔV_3				.39	.44	6.98	5.96	2.87	2.54
ΔV Translunar	65.1	35.6	39.6	24.57	17.63	32.25	26.14	26.80	20.36
ΔV_4				4.78	3.58	4.78	3.58	4.78	3.58
ΔV_5				.47	.19	12.08	16.12	1.35	1.78
ΔV_6				.12	.13	41.87	41.88	4.53	4.53
ΔV Transearth	83.6	27.9	12.2	5.37	3.90	58.73	61.59	10.65	9.90
ΔV Total	148.7	63.5	51.8	29.94	21.53	90.98	87.73	37.45	30.26
δR Perilune (ft.)	22,700.	20,100.	30,600.	37.	25.	1,498.	1,463.	703.	644.
δV Perilune (ft/sec)	46.7	24.1	26.4	3.30	2.22	7.98	7.74	4.31	3.87
δR Perigee (ft.)	33,300.	14,300.	6,870.	7.	7.	7,087.	7,088.	639.	768.
δV Perigee (ft/sec)	81.3	41.5	33.3	12.56	1.25	45.78	54.31	13.15	6.00

APPENDIX

Development of Midcourse ΔV Computations

The analysis of this paper is based on a linearized representation of deviations from a nominal trajectory. Define a state vector consisting of the position and velocity vectors;

$$\underline{x} = \begin{Bmatrix} \underline{r} \\ \underline{v} \end{Bmatrix} \tag{1}$$

The differential equation governing $\underline{x}(t)$ is;

$$\dot{\underline{x}} = \begin{Bmatrix} \underline{v} \\ \mu \underline{r}/r^3 + \underline{g}_p \end{Bmatrix} \tag{2}$$

where $\underline{g}_p$ is perturbing accelerations due to bodies other than the reference body. If $\underline{x}$ is varied, a first order perturbation can be generated using the equation

$$\begin{aligned} \delta\dot{\underline{x}} &= \nabla_x(\dot{\underline{x}})\,\delta\underline{x} \\ &= F\,\delta\underline{x} \end{aligned} \tag{3}$$

where, in partitioned form,

$$F = \begin{bmatrix} 0 & I \\ G & 0 \end{bmatrix}$$

$$G = -\sum_j \frac{\mu_j}{r_{jv}^3}\left(I - \frac{3}{r_{jv}^2}\underline{r}_{jv}\underline{r}_{jv}^T\right)$$

and $\underline{r}_{jv}$ is the position vector from an attracting body to the vehicle, μ_j the associated gravitational parameter.

For the linear deviations, it is now possible to define a transition matrix, ϕ, where

$$\begin{aligned} \phi(t_o) &= I \\ \dot{\phi}(t) &= F\phi(t) \end{aligned} \tag{4}$$

This matrix has the property of updating the deviation state,

$$\delta\underline{x}(t) = \phi(t,t_o)\,\delta\underline{x}(t_o) \tag{5}$$

where $\phi(t,t_o) = \int_{t_o}^{t} \dot{\phi}\,dt, \phi(t_o) = I.$

For convenience in the work to follow, define the partitioned ϕ matrix.

$$\phi(t,t_0) = \begin{bmatrix} \phi_1(t,t_0) & \phi_2(t,t_0) \\ \phi_3(t,t_0) & \phi_4(t,t_0) \end{bmatrix} \tag{6}$$

The guidance problem in this linearized model consists of extrapolating deviations up to a correction time (t_c), then solving for the required velocity to null the position deviations at the terminal time (T). Then

$$\begin{Bmatrix} \delta \underline{r}(t_c^-) \\ \delta \underline{v}(t_c^-) \end{Bmatrix} = \phi(t_c,t_0) \begin{Bmatrix} \delta \underline{r}(t_0) \\ \delta \underline{v}(t_0) \end{Bmatrix} \tag{7}$$

where (t_c^-) denotes deviations prior to the correction. Since we are not free to manipulate the position deviation,

$$\delta \underline{r}(t_c^-) = \delta \underline{r}(t_c^+) \tag{8}$$

Now the terminal conditions are

$$\begin{Bmatrix} \delta \underline{r}(T) \\ \delta \underline{v}(T) \end{Bmatrix} = \phi(T,t_c) \begin{Bmatrix} \delta \underline{r}(t_c^+) \\ \delta \underline{v}(t_c^+) \end{Bmatrix} \tag{9}$$

and the boundary condition is

$$\delta \underline{r}(T) = \underline{0} \tag{10}$$

Combining (6), (9), and (10), we find that

$$\underline{0} = \phi_1(T,t_c)\,\delta \underline{r}(t_c^+) + \phi_2(T,t_c)\,\delta \underline{v}(t_c^+)$$

Hence,

$$\delta \underline{v}(t_c^+) = \phi_2(T,t_c)^{-1}\phi_1(T,t_c)\,\delta \underline{r}(t_c^-) \tag{11}$$

Accordingly, we can calculate the required velocity increment at a correction time by

$$\begin{aligned} \Delta \underline{v}(t_c) &= \delta \underline{v}(t_c^+) - \delta \underline{v}(t_c^-) \\ &= \phi_2(T,t_c)^{-1}\phi_1(T,t_c)\,\delta \underline{r}(t_c^-) - \delta \underline{v}(t_c^-) \end{aligned} \tag{12}$$

Now for our purposes, we are concerned with depicting the covariance matrix of our errors. This matrix is defined as

$$X(t) = E\left\{ \delta x(t)\, \delta x(t)^T \right\} \tag{13}$$

where the E operator denotes the mathematical expectation. Extrapolation of the x matrix is evidently accomplished by

$$X(t) = \phi(t,t_o)\, X(t_o)\, \phi(t,t_o)^T \tag{14}$$

Again, let us partition x for convenience sake as

$$X(t) = \begin{bmatrix} X_1(t) & X_2(t) \\ X_3(t) & X_4(t) \end{bmatrix}$$

The covariance matrix for a velocity correction can be defined as

$$V(t_c) = E\left\{ \Delta v(t_c)\, \Delta v(t_c)^T \right\}$$

if we rewrite (12) as

$$\Delta v(t_c) = A\, \delta x(t_c^-)$$

where $A = \left[\left(\phi_2(T,t_c)^{-1} \phi_1(T,t_c) \right) - I \right]$

Then,

$$\begin{aligned} V(t_c) &= E\left\{ A\, \delta x(t_c^-)\, \delta x(t_c^-)^T A^T \right\} \\ &= A\, X(t_c^-)\, A^T \end{aligned} \tag{15}$$

The RSS magnitude of the velocity correction is derived from

$$\sigma_{\Delta v(t_c)} = \left(TR\ V(t_c) \right)^{1/2} \tag{16}$$

where $\sigma_{\Delta v(t_c)}$ is the variance in correction for our linearized system.

The remaining problem is representation of residual errors from a velocity correction. It is assumed that velocity corrections are applied impulsively, and hence that no additional position errors accrue during corrections. Two types of velocity correction errors were considered. The first consisted of an error in magnitude which is directly proportional to the correction magnitude and was developed by Battin of MIT in his R-341. The second assumes a magnitude error with constant variance independent of the correction magnitude, and is developed below.

Assume that a transformation matrix M exists such that the following expression is true;

$$\Delta \hat{\underline{v}} = \Delta \hat{v}\, M \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} \tag{17}$$

where $\Delta\hat{v}$ is the desired velocity correction. The actual velocity correction is subject to an additive incremental error in magnitude, δv, and an offset angle γ in some polar orientation β. If γ is assumed to be small, then the actual velocity correction can be approximated by

$$\Delta \underline{v} \approx (\Delta\hat{v} + \delta v)\, M \begin{Bmatrix} \gamma\cos\beta \\ \gamma\sin\beta \\ 1 \end{Bmatrix} \tag{18}$$

The error in the velocity correction is written

$$\begin{aligned} \underline{\eta} &= \Delta\hat{\underline{v}} - \Delta\underline{v} \\ &\approx -\gamma(\Delta\hat{v} + \delta v)\, M \begin{Bmatrix} \cos\beta \\ \sin\beta \\ 0 \end{Bmatrix} - \frac{\delta v}{\Delta\hat{v}}\, \Delta\hat{\underline{v}} \end{aligned} \tag{19}$$

Assume that β is independent and uniformly distributed over 2π, and the γ and δv are independent random variables with zero mean and mean squared values $\overline{\gamma^2}$ and $\overline{\delta v^2}$. Finally, the covariance of this error, expressed as a perturbation to the fourth partition of $X(t_c)$, is

$$\begin{aligned} \Delta X_4(t_c) &= E\{\eta\eta^T\} \\ &= \overline{\gamma^2}\,(\mathrm{Tr}\,V(t_c) + \overline{\delta v^2})\, I \\ &\quad + \left(\overline{\delta v^2}(1-\overline{\gamma^2})/\mathrm{Tr}\,V(t_c) - \overline{\gamma^2}\right) V(t_c) \end{aligned} \tag{20}$$

In essentially the same manner, Battin's error representation is

$$\Delta \underline{v} = (1 + K)\, \Delta\hat{v}\, M \begin{Bmatrix} \gamma\cos\beta \\ \gamma\sin\beta \\ 1 \end{Bmatrix} \tag{21}$$

where K is a random scale factor for the correction. For this type of error,

$$\begin{aligned} \Delta X_4(t_c) &= \frac{\overline{\gamma^2}}{2}\,\mathrm{Tr}\,V(t_c)\, I \\ &\quad + \left(\overline{K^2} - \frac{\overline{\gamma^2}}{2}\right) V(t_c) \end{aligned} \tag{22}$$

The above development contains all the elements required to analyze the correction requirements for a mission. To summarize the calculation procedure, first a set of correction times is selected, and transition matrices bridging them and extrapolating from each correction time to the terminal time are generated. The covariance matrix of initial errors is then propagated to the first correction time as in (14), and the required velocity correction calculated by (15). The correction is added to $\bar{X}$ along with the associated correction execution error from either (20) or (22). The new $\bar{X}$ matrix is then propagated to the next correction time and the process is repeated. After the final correction, the $\bar{X}$ matrix is propagated to the terminal time to determine the variances in position and velocity at that point, using the relation

$$\delta x_j(T) = \left(\bar{X}(T)_{jj}\right)^{1/2} \tag{23}$$

where δx_j is the j^{TH} component of the variance state vector.